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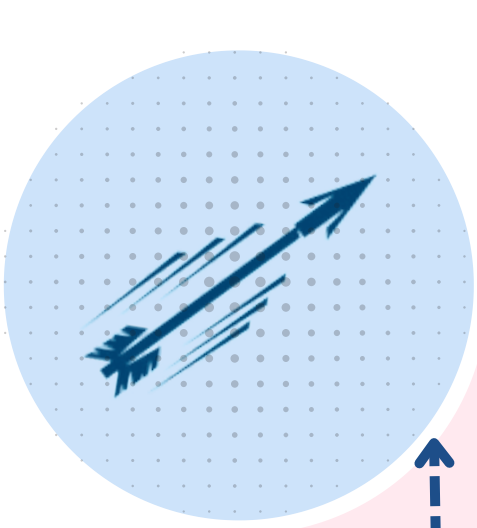
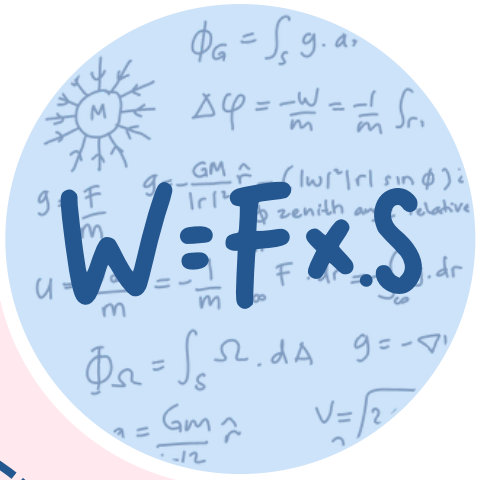
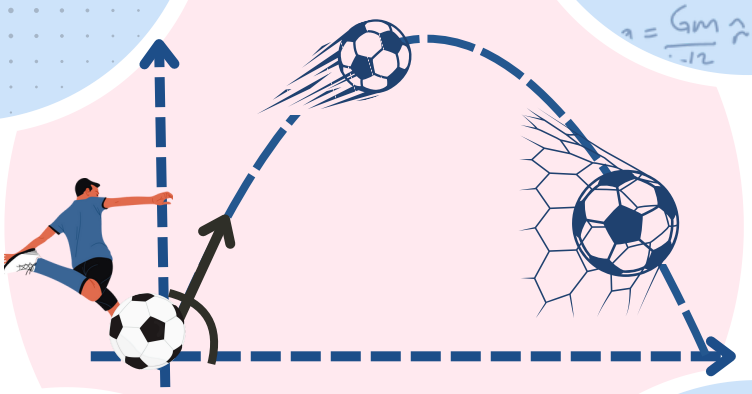
MOTION IN TWO DIMENSION







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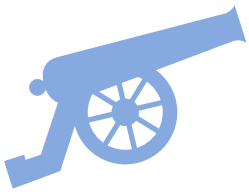
NOTES
MOTION IN TWO DIMENSIONS

PROJECTILE MOTION

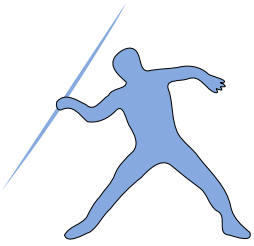
- The form of 2-dimensional motion experienced by an object or a particle that is thrown near Earth's Surface and travels a curved path under the action of gravity only
- (In particular the effects of air resistance are assumed to be negligible).
- A cricket or a football hit into the air, a shell fired from a cannon or a rock thrown down a hill are all examples of Projectile motion.

Examples of Projectile Motion:

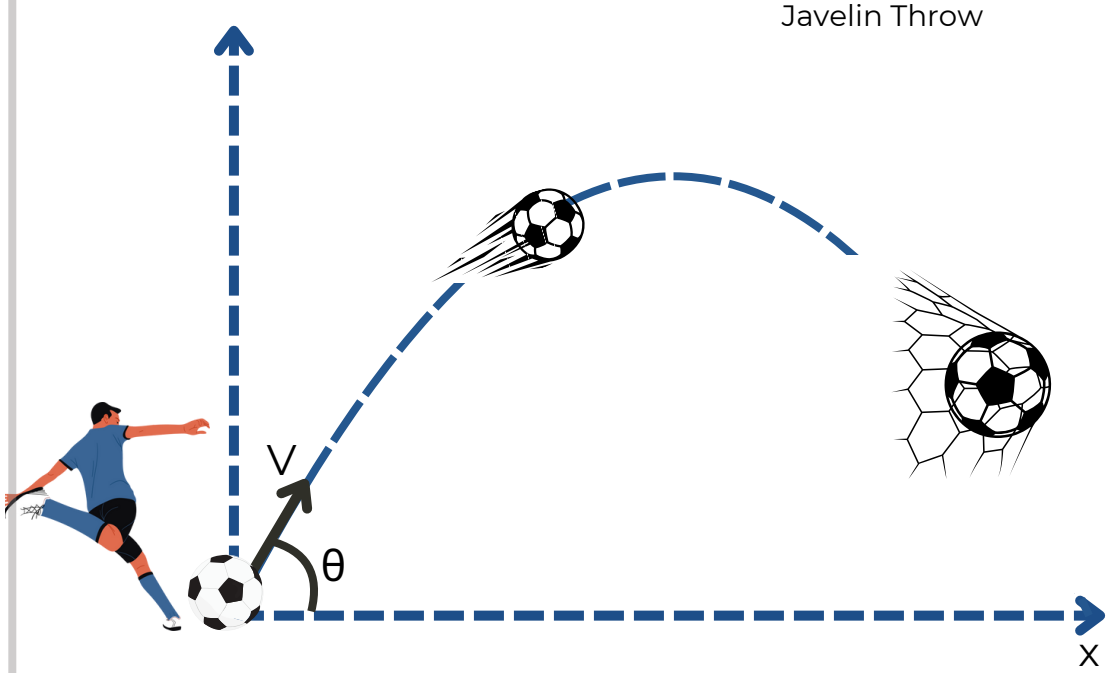
Throwing Ball



Canon Ball



Javelin Throw



PROJECTILE TRAJECTORY

- The path followed by a Projectile is called the trajectory.
- Projectile motion only occurs when there is one force applied at the beginning of the trajectory after which no force apart from gravity Is applied.
- The vertical component of acceleration has a magnitude of 9.80ms/2. While the horizontal component has a magnitude of 0 ms/2.
- Ideal Projectile motion neglects air resistance and wind speed, spin of the projectile, and other effects influencing the flight of a real life projectile.

Velocity:

- Consider a Projectile which is thrown with Velocity vo making an angle θ with the horizontal X-Axis.
- The horizontal component of velocity remains constant while the vertical component of velocity changes uniformly and becomes zero at the highest point.
- To find velocity of the body at any particular time we must know the x and y components at that point:
- By first Equation of motion along x-axis:

$$V_{Px} = V_{ix} + a_x t$$

Where, $V_{Px} = V_x$

$V_{ix} = V_o \cos\theta$

$a_x = 0$

$t = t$

Thus from This we obtain that;

$V_x = V_o \cos\theta$ —(i)

- By first Equation of motion along y-axis;

$$V_{Py} = V_{iy} + a_x t$$

Where, $V_{Py} = V_P$

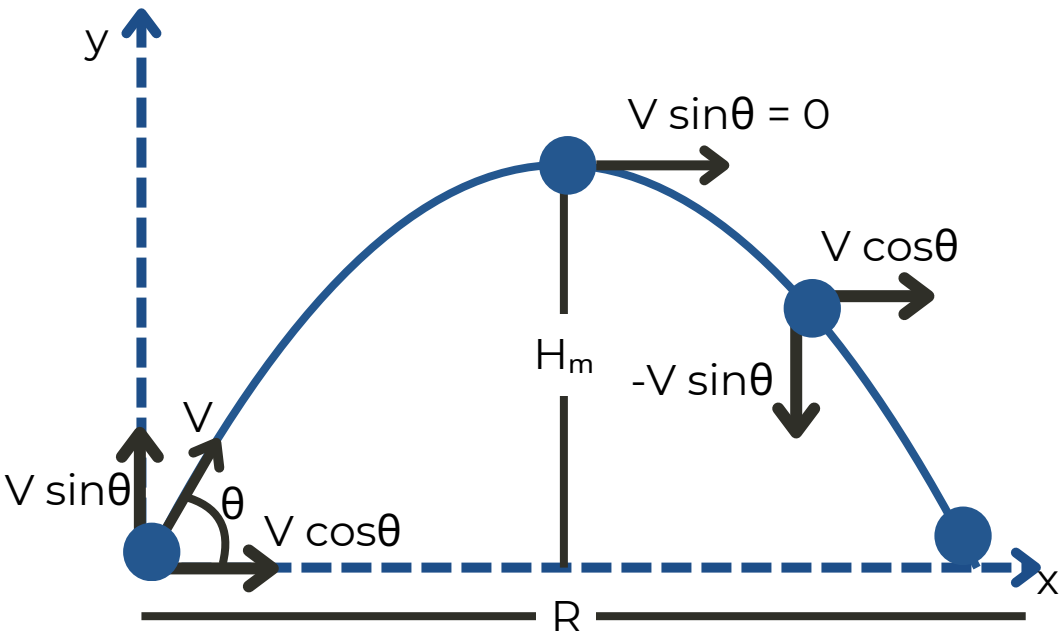
$V_{iy} = V_o \sin\theta$

$a_y = -g$

$t = t$

Thus from This we obtain that;

$V_y = V_o \sin\theta - gt$ —(ii)



NOTES

- Thus by rectangular components we can find out the magnitude by using this formula:

$$V = \sqrt{(V_x^2 + V_y^2)}$$

- Putting values of Equation (i) and Equation (ii):

$V = \sqrt{(V_o \cos\theta)^2 + (V_o \sin\theta - gt)^2}$

- -Direction can also be found out using rectangular components;

$$\theta = \tan^{-1} (V_x / V_y)$$

- Putting values of Equation (i) and Equation (ii):

$\theta = \tan^{-1} (V_o \sin\theta - gt / V_o \cos\theta)$

Time of flight:

- Time taken by a projectile to go from point of projection to the point of impact is called time of flight.
- Consider a Projectile which is thrown with Velocity vo making an angle θ with the horizontal X-Axis. To find time of flight we use second Equation of motion:

$$S_y = V_{iy}t + 1/2\ a_y\ t2$$

Where $S_y = 0$

$V_{iy} = V_o \sin\theta$

$a_y = -g$

$t = T$

- Thus from this we obtain that;

$T = 2V_o \sin\theta / g$

- Time to reach the summit(highest Point) will be half of the time of flight. Thus :

$T' = T / 2$

Range:

- The horizontal distance from the point of projection to the point of impact is called the range of projectile.
- Consider a projectile which is thrown with velocity vo making an angle θ with the horizontal x-Axis. To find range we use second Equation of motion:

$$S_x = V_{ix}\ t + 1/2\ a_{ix}\ t$$

Where $S_x = R$

$V_{ix} = V_o \cos\theta$

$a_x = 0$

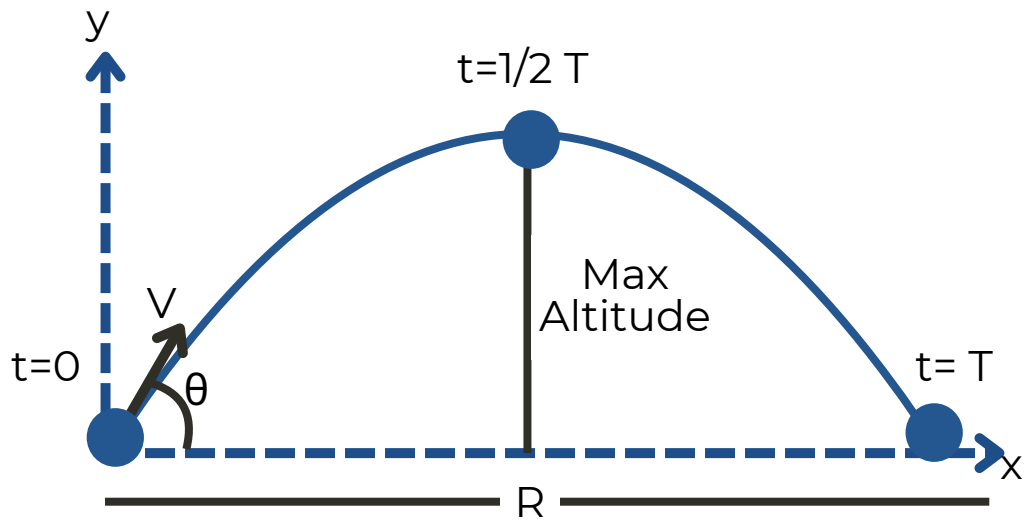
$t = T = 2V_o \sin\theta / g$

- Thus from This we obtain that;

$R = V_o \cos\theta\ (\ 2V_o \sin\theta / g\ ) + 0$

- Further solve to obtain the following Equation:

$R = V_o^2 \sin 2\theta / g$



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MAXIMUM HEIGHT AND RANGE OF PROJECTILE

Maximum Height:

- Maximum vertical distance reached by a projectile form projection level is called maximum Height.
- Consider a Projectile which is thrown with Velocity V_o making an angle θ with the horizontal x-Axis. To find maximum height of the body we use third Equation of motion:

$$2a_y S = V_{fy}^2 - V_{iy}^2 \quad \text{—(iii)}$$

Where $a_y = -g$

$S = H$

$V_{fy} = V_y = 0$

$V_{iy} = V_o \sin\theta$

- Thus from This we obtain that;

$$-2gH = (0) - (V_o \sin\theta)^2$$

$$\mathbf{H = V_o \sin^2\theta / 2g}$$

Maximum range:

- Since the maximum value for the sine of any angle is 1, so the factor sin 20 will be maximum if it is equal to 1 as well, so:

$$\sin 2\theta_{max} = 1$$

$$\text{Or } 2\theta = \sin^{-1}(1)$$

$$\text{Since, } 2\theta = 90°$$

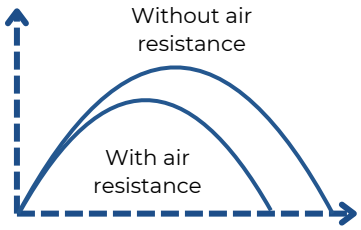
$$\text{So, } \mathbf{\theta = 45°}$$

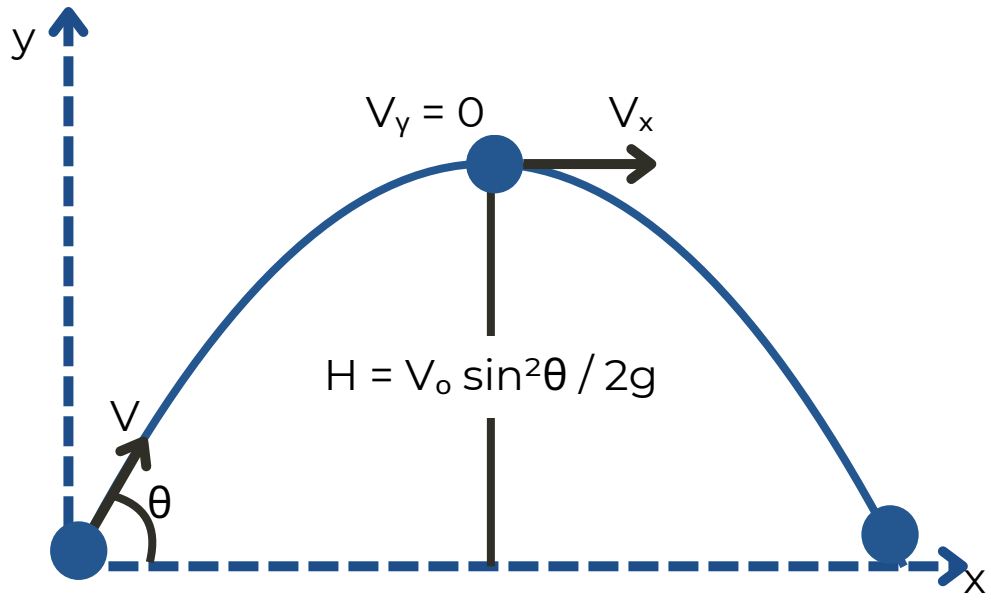
- Therefore when an object is projected at an angle of 45° the range will be maximum.

$$\mathbf{R_{max} = V_o^2 / g}$$

Range with air resistance:

- Air resistance affects both the horizontal and the vertical component of velocity and hence
- the range of Projectile is reduced.





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Two Projectile angles for the same range:

- If the velocity of projection v, and the acceleration due to gravity g is kept constant, then there are two complementary angles (the sum of angles makes 90°) will have the same horizontal range:
- For example the range at 75°& 15° is the same:

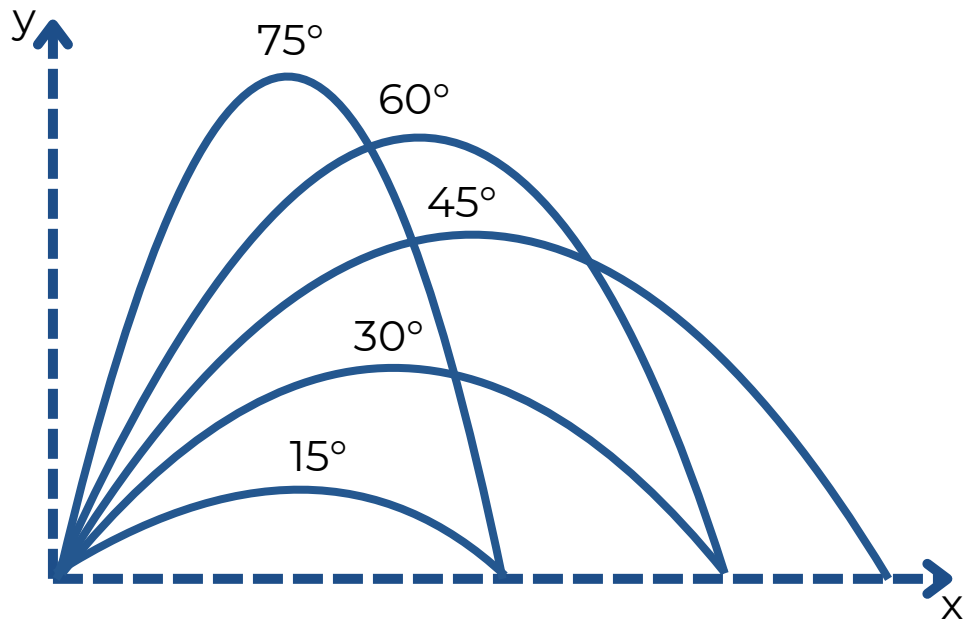
  $R_{75} = V_o^2 \sin 2(75°)$
- Upon solving:

  $R_{75} = V_o^2 \sin (0.5)$
- Now:

  $R_{15} = V_o^2 \sin 2(15°)$
- Upon solving:

  $R_{15} = V_o^2 \sin (0.5)$
- Hence, the range at 75° & 15° is the same.
- Similarly the range 60° and 30° is the same.
- Similarly for any 2 such angles (equal degrees above and below 45°) we can show that the range is the same.

Range	$R = V_o^2 \sin 2\theta / g$
Maximum Height	$H = V_o \sin^2\theta / 2g$
Time	$T = 2V_o \sin\theta / g$
Velocity	$V = \sqrt{(V_o \cos\theta)^2 + (V_o \sin\theta - gt)^2}$
Direction	$\theta = \tan^{-1}(V_o \sin\theta - gt / V_o \cos\theta)$
Maximum Range	$R_{max} = V_o^2 / g$



NOTES

APPLICATION OF PROJECTILE MOTION

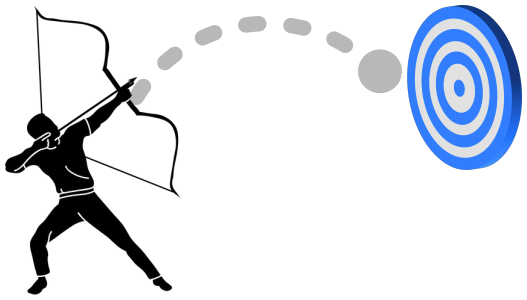
A ballistic flight is that in which a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity.

An unpowered and un-guided missile is called a ballistic missile and the path followed by it is called ballistic trajectory.

A ballistic missile moves in a way that is the result of the superposition of two independent motions: a straight line inertial flight in the direction of the launch and a vertical gravity fall. By law of inertia, an object should sail straight off in the direction thrown, at constant speed equal to its initial speed particularly in empty space. But the downward force of gravity will alter the straight path into a curved trajectory.

For short ranges and flat Earth approximation, the trajectory is parabolic but the dragless ballistic trajectory for spherical Earth should actually be elliptical. At high speed and for long trajectories the air friction is not negligible and sometimes the force of air friction is more than gravity. It affects both horizontal as well as vertical motions. Therefore, it is completely unrealistic to neglect the aerodynamic forces.

- The shooting of a missile on a selected distant spot is a major element of warfare. It undergoes complicated motions due to air friction and wind etc.
- Consequently the angle of projection can not be found by the geometry of the situation at the moment of launching. The actual flights of missiles are worked out to high degrees of precision and the results were contained in tabular form. The modified equation of trajectory is too complicated to be discussed here.
- The ballistic missiles are useful only for short ranges. For long ranges and greater precision, powered and remote control guided missiles are used.



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